Selecting the Best Distillation Control Configuration

The selection of an appropriate control configuration (structure) is the most important decision when designing distillation control systems. The steady state RGA is commonly used in industry for selecting the best structure. One counterexample to the usefulness of this measure is the DB configuration, which has infinite steady state relative gain array (RGA) values, but still good control performance is possible. This is indicated by high-frequency RGA values close to 1. In this paper it is stressed that decisions regarding controller design should be based on the initial response (high-frequency behavior) rather than the steady state.

Based on a frequency-dependent RGA analysis and optimal PI controller designs of four different configurations, the (L/D) (V/B) configuration is found to be the best choice for two-point composition control. The conventional LV configuration performs poorer than the above system, but is preferable if one-point control is used.

Sigurd Skogestad
Petter Lundström
Elling W. Jacobsen
Chemical Engineering
Norwegian Institute of Technology (NTH)
N-7034 Trondheim, Norway

Introduction

Consider the simple two-product distillation column shown in Figure 1. At steady state, the column has two degrees of freedom, which may be used, for example, to specify top and bottom compositions (y_D and x_B). However dynamically the levels and pressure may vary, from a control point it may be viewed as a 5×5 system. The optimal controller should, based on all available information (measurements, process model, expected disturbances) manipulate *all* five inputs (L, V, V_T, D, B) in order to keep the five outputs (levels in top and bottom, pressure, top and bottom composition) as close as possible to their desired values. In practice few columns, if any, are controlled using a full 5×5 controller. Instead a decentralized system with single-loop controllers is used. This kind of system is easier to understand and retune, is more failure tolerant, and is less sensitive to plant operation. Since the levels and pressure must be controlled at all times to ensure stable operation, the level and pressure loops are designed first. Engineers often do a good job in designing this subsystem reliably, but many fail to recognize the profound influence the choice of level controllers has on the remaining 2×2 composition control problem.

As an example, consider the conventional choice of controlling pressure with cooling V_T, top level with distillate D, and bottom level with bottoms flow B. This gives rise to the LV configuration, shown in Figure 2. It is given this name because the reflux L and boilup V are the remaining independent

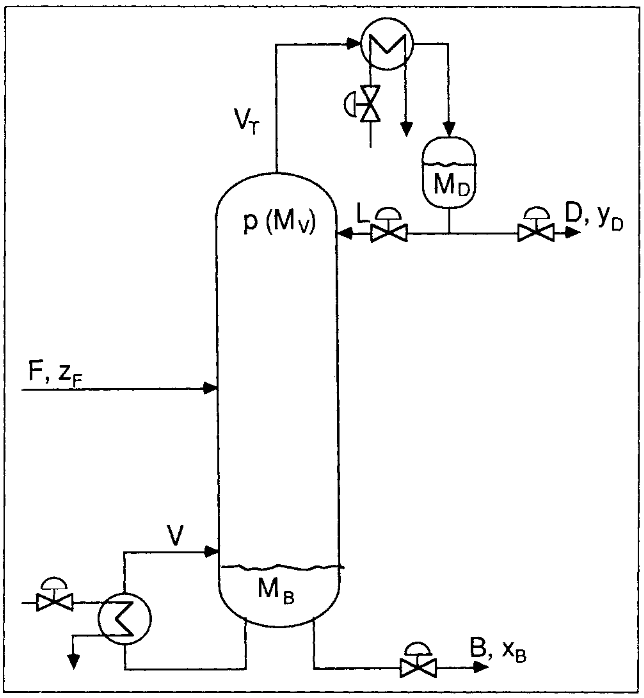

Figure 1. Two-product distillation column with single feed and total condenser.

variables to be used for composition control. The DV configuration, with completely different characteristics, is obtained if L is used to control the top level instead of D. For example, whereas the compositions in the DV configuration are only weakly sensitive to disturbances in boilup or reflux, the LV configuration is very sensitive to such changes. In other words, the action of the level loops causes some configurations to have better built-in rejection of disturbances than others. In particular, the double ratio configurations—for example, the (L/D) (V/B) configuration—are good in this respect.

In this paper three different modes of operation are considered with respect to the difference between the control configurations:

1. Open-loop (manual) operation
2. One-point composition control
3. Two-point composition control

The first mode (open loop) is not too important in itself, but it is of interest for the other two modes because low open-loop sensitivity to disturbances (good built-in disturbance rejection) introduces less need for feedback control and makes the remaining control problem simpler.

Although most industrial columns are operated with one-point control, the emphasis here will be on two-point control. The reason is that this is the most difficult problem and has potential for economic savings. Furthermore, even when simple economic considerations of the column itself does not warrant two-point control, avoiding variations in compositions may prove worthwhile because it yields less variations in downstream units and a more uniform quality of other final products.

This paper addresses the important issue of control configuration selection for distillation columns, that is, which two independent variables to use for composition control. The issue has been discussed for quite some time by industrial people (Shinskey, 1967, 1984; McCune and Gallier, 1973), but has only recently been treated rigorously by academic researchers

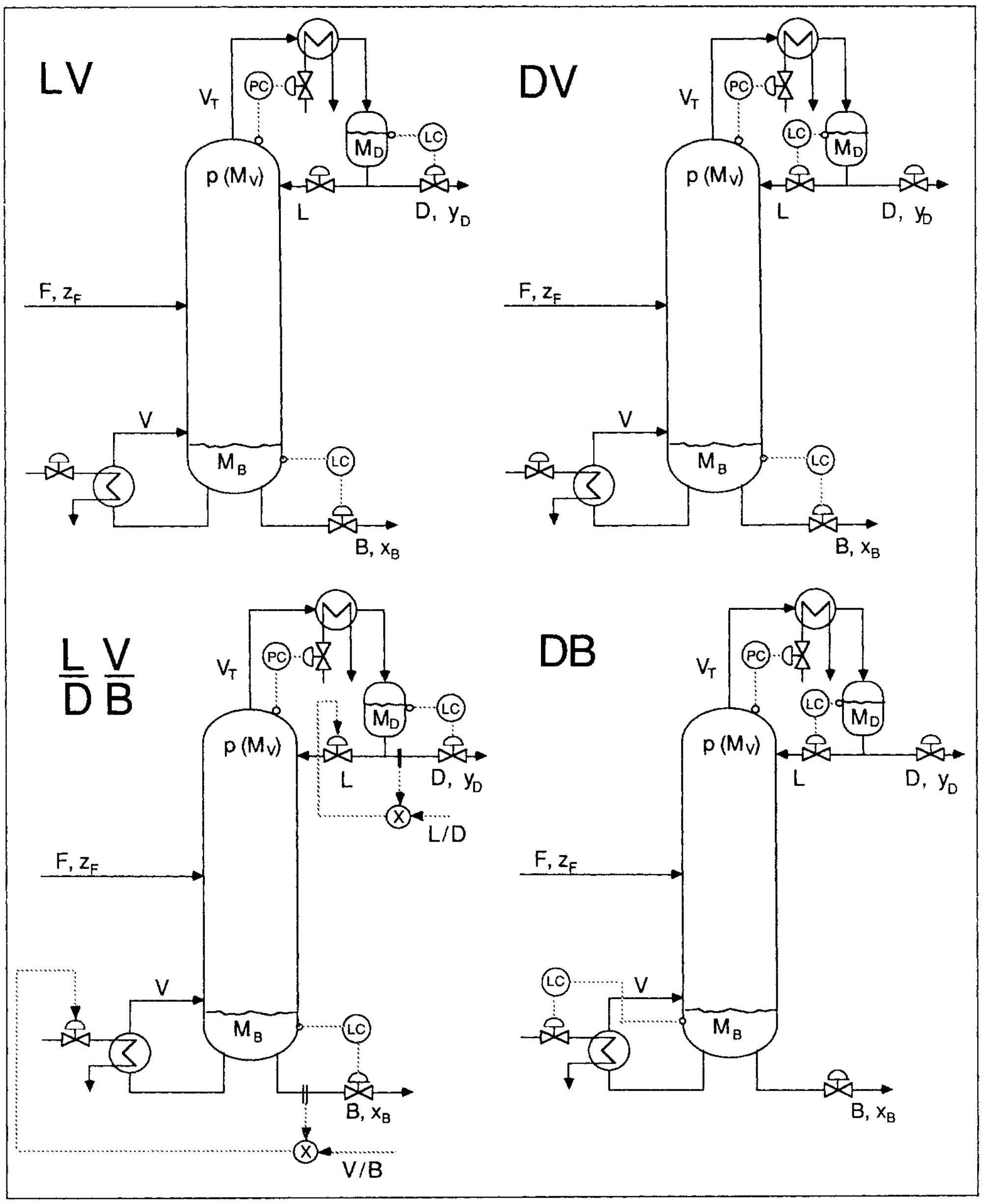

Figure 2. Control of distillation column with four different configurations.

Table 1. Steady-state Data for Distillation Column Examples

Column	z_F	α	N	N_F	$1 - y_D$	x_B	D/F	L/F
A	0.5	1.5	40	21	0.01	0.01	0.500	2.706
B	0.1	1.5	40	21	0.01	0.01	0.092	2.329
C	0.5	1.5	40	21	0.10	0.002	0.555	2.737
D	0.65	1.12	110	39	0.005	0.10	0.614	11.862
E	0.2	5	15	5	0.0001	0.05	0.158	0.226
F	0.5	15	10	5	0.0001	0.0001	0.500	0.227
G	0.5	1.5	80	40	0.0001	0.0001	0.500	2.635

All columns have liquid feed ($q_F = 1$)

(Waller, 1986; Takamatsu et al., 1987; Skogestad and Morari, 1987a). In many ways, this paper attempts to answer some of the questions raised by Skogestad and Morari (1987a).

Because of the large number of possible configurations for a given column, there is clearly a need for tools that may assist the engineer in making the best selection. Luyben (1979, 1989) emphasizes the large diversity of columns, processes, and plants, and seems to doubt that a simple tool may be found. This is partly supported by a recent paper by Birky et al. (1989). They compared the rules of Page Buckley and Greg Shinskey, who both are well-known industrial experts on distillation column control, on a set of example columns and found that they agreed in only three out of 18 cases. There may be a number of reasons for these differences, but the most important one is probably that Buckley considers mostly level control and one-point composition control, whereas Shinskey also addresses two-point control. Nevertheless, the above results clearly demonstrate the need for better understanding and for developing improved tools.

Shinskey's rule is to choose a configuration with a relative gain array (RGA) value in the range from about 0.9 to 4 (Shinskey, 1984), and this rule seems to be widely used in industry. RGA value here means $\lambda_{11}(0)$, that is, the steady state value of the 1,1 element of the relative gain array. The RGA is easily computed from the gain matrix for a given configuration. However, the steady state RGA contains no information about disturbances and dynamic behavior, both of which are crucial for evaluating control performance. The fact that this measure has proved to be very useful for distillation columns must therefore be a result of fortunate circumstances. Another objective of this paper is to investigate this in more detail.

One counterexample which demonstrates that the steady state RGA is generally *not* reliable is the DB configuration. It involves using distillate product *D* and bottom product *B* for composition control. This control scheme has previously been labeled impossible by most distillation experts (Shinskey, 1984, p. 154) because its RGA is infinite at steady state. Yet, Finco et al. (1989) have shown both with simulations and with actual implementation that the scheme is indeed workable. Skogestad et al. (1990) have investigated in more detail why the DB configuration works, and we shall use some of their results in this paper. The most important conclusion is that the initial response is of primary importance for feedback control. This implies that the tools we choose to use for selecting control configurations must reflect, to some degree, the behavior of the initial response.

Example columns

Throughout the paper we shall make use of the seven example columns, A–G, studied by Skogestad and Morari (1988a). Steady state data for these columns are given in Table 1. For all examples, we assume constant molar flows, binary mixtures, constant relative volatility, negligible vapor holdup, and perfect control of pressure and levels. However, in contrast to Skogestad and Morari (1988a) we will include the liquid flow dynamics in the model. The steady state holdup on all trays, including reboiler and condenser, is chosen as $M_i/F = 0.5$ min.

The steady state gain matrix for the LV configuration is shown in Table 2 for all columns. Note that scaled (relative, logarithmic) compositions have been used:

$$\Delta y_D^S = \Delta y_D/(1 - y_D);$$
$$\Delta x_B^S = \Delta x_B/x_B \tag{1}$$

Table 2 also shows $\lambda_{11}^{LV}(0)$ and the time constants τ_1, τ_2, and θ_L, which are used in the simplified dynamic model discussed below. θ_L is the overall liquid lag from the top to the bottom of the column.

The following configurations are considered in the paper:

- LV, often denoted "energy balance" or "indirect material balance"
- DV, material balance
- DB,
- (L/D)V, "Ryskamps scheme"
- (L/D)(V/B), double ratio.

Table 2. Data Used in Simple Model of Distillation Columns, Eq. 8

Column	$G^{LV}(0)^S$	$\lambda_{11}^{LV}(0)$	τ_1 min	τ_2 min	θ_L min
A	$\begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix}$	35.1	194	15	2.46
B	$\begin{bmatrix} 174.79 & -171.7 \\ 90.191 & -90.5 \end{bmatrix}$	47.5	250	15	2.86
C	$\begin{bmatrix} 16.023 & -16.0 \\ 9.29 & -10.7 \end{bmatrix}$	7.53	24	10	2.44
D	$\begin{bmatrix} 24.585 & -24.2 \\ 21.270 & -21.3 \end{bmatrix}$	58.7	154	30	1.54
E	$\begin{bmatrix} 203.4 & -131.5 \\ 22.47 & -22.5 \end{bmatrix}$	2.82	82	30	11.06
F	$\begin{bmatrix} 10{,}740 & -10{,}730 \\ 9{,}257 & -9{,}267 \end{bmatrix}$	499	2,996	4	7.34
G	$\begin{bmatrix} 8{,}648.94 & -8{,}646 \\ 11{,}347.06 & -11{,}350 \end{bmatrix}$	1,673	20,333	30	5.06

Modeling

Full-order model

With the above assumptions we may easily derive a full-order model with two states on each stage (composition of light component and liquid holdup). A simple linear relationship between liquid flow and holdup is used: $L_i(t) = L_i^o + (M_i(t) - M_i^o)/\tau_L$. Here the hydraulic lag on all trays is $\tau_L = \theta_L/(N - 1)$. This nonlinear model is used in all the simulations.

Simplified linear model

Because of the large number of trays in some of the columns (110 trays for column D, which give a 220th-order full model) we choose to use the simple two time-constant dynamic model presented by Skogestad and Morari (1988a) as the basis for controller design. Using a simple model also makes it easier for the reader to interpret and check the results. The two time-constant model is derived assuming the flow and composition dynamics to be decoupled, and then the two separate models for the composition and flow dynamics are simply combined. In reality, the flow dynamics do affect the composition dynamics and the model will be somewhat in error. In particular, the time constant τ_2 may be different (larger) than the one shown in Table 2, which was obtained by Skogestad and Morari (1988a) by neglecting flow dynamics. However, the results in this paper appear to be only weakly dependent on τ_2.

Let $dL_T \equiv dL$ and dV_T represent small changes in the liquid and vapor flows in the top of the column, and let dL_B and $dV_B \equiv dV$ be changes in the bottom. In addition to constant molar flows we make the following assumptions:

1. Immediate vapor response (perfect pressure control)

$$dV_T(s) = dV(s) \tag{2}$$

2. Vapor flow has no effect on holdup and liquid flow from top to bottom may be modeled as n lags in series

$$dL_B(s) = g_L(s)dL(s) \tag{3}$$

where

$$g_L(s) = \frac{1}{[1 + (\theta_L/n)s]^n} \tag{4}$$

Strictly speaking, n should be equal to the number of trays in the column, N-1, but is here chosen as $n = 5$ for all columns in order to reduce the order of the model.

3. Perfect control of reboiler and condenser level

$$dL_T(s) \equiv dL(s) = dV_T(s) - dD(s) \tag{5}$$

$$dV_B(s) \equiv dV(s) = dL_B(s) - dB(s) \tag{6}$$

To generalize these equations to the case of not-constant molar flows and to the case when changes in vapor flow affect holdups, additional parameters must be introduced in Eqs. 2 and 3.

LV Configuration. The simplified model of the LV configuration then becomes

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = G^{LV}(s) \begin{pmatrix} dL \\ dV \end{pmatrix} \tag{7}$$

$$G^{LV}(s) = \begin{bmatrix} \dfrac{k_{11}}{1 + \tau_1 s} & \left(\dfrac{k_{11} + k_{12}}{1 + \tau_2 s} - \dfrac{k_{11}}{1 + \tau_1 s}\right) \\ \dfrac{k_{21}}{1 + \tau_1 s} g_L(s) & \left(\dfrac{k_{21} + k_{22}}{1 + \tau_2 s} - \dfrac{k_{21}}{1 + \tau_1 s}\right) \end{bmatrix} \tag{8}$$

where $k_{ij} = g_{ij}^{LV}(0)$ denote the steady state gains for the LV configuration. This model is used throughout the paper unless otherwise stated.

Linear models for other configurations

Several authors (Häggblom and Waller, 1988; Skogestad and Morari, 1987b) have discussed how to obtain steady state models for various configurations, but their transformations do not include the flow dynamics that are crucial for the dynamic behavior.

Let u_i represent an arbitrary independent variable used for composition control. A dynamic model for the u_1u_2 configuration is easily obtained from Eq. 8 by expressing dL and dV in terms of u_1 and u_2 using Eqs. 2 to 6. This gives

$$\begin{pmatrix} dL \\ dV \end{pmatrix} = M_{LV}^{u_1u_2}(s) \begin{pmatrix} du_1 \\ du_2 \end{pmatrix} \tag{9}$$

and the linear model for the new configuration becomes

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = G^{u_1u_2}(s) \begin{pmatrix} du_1 \\ du_2 \end{pmatrix} \tag{10}$$

$$G^{u_1u_2}(s) = G^{LV}(s) M_{LV}^{u_1u_2}(s) \tag{11}$$

Takamatsu et al. (1987) derive similar expressions, but use different notation. If ratios are used as independent variables, these must first be expressed as a function of the actual flows, for example with $u_i = L/D$,

$$d(L/D)(s) = (1/D)dL(s) - (L/D^2)dD(s) \tag{12}$$

DV Configuration. With constant molar flows, immediate vapor response, and perfect level control we have $dL = -dD + dV$, that is,

$$M_{LV}^{DV}(s) = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \tag{13}$$

DB configuration. We have $dV = g_L(s)dL - dB$ where $dL = dV - dD$. Eliminating dV yields $dL\,[1 - g_L(s)] = -dD - dB$ and $dV[1 - g_L(s)] = -g_L(s)dD - dB$. We obtain

$$M_{LV}^{DB}(s) = \frac{1}{1 - g_L(s)} \begin{bmatrix} -1 & -1 \\ -g_L(s) & -1 \end{bmatrix} \tag{14}$$

Note that

$$\lim_{s \to 0} \frac{1}{1 - g_L(s)} = \frac{1}{\theta_L s} \tag{15}$$

and $\lim_{s \to 0} g_L(s) = 1$. The elements in the gain matrix for the DB configuration will therefore approach infinity at low frequency (steady state) and it will also become singular. The physical interpretation is that a decrease in, for example, D with B

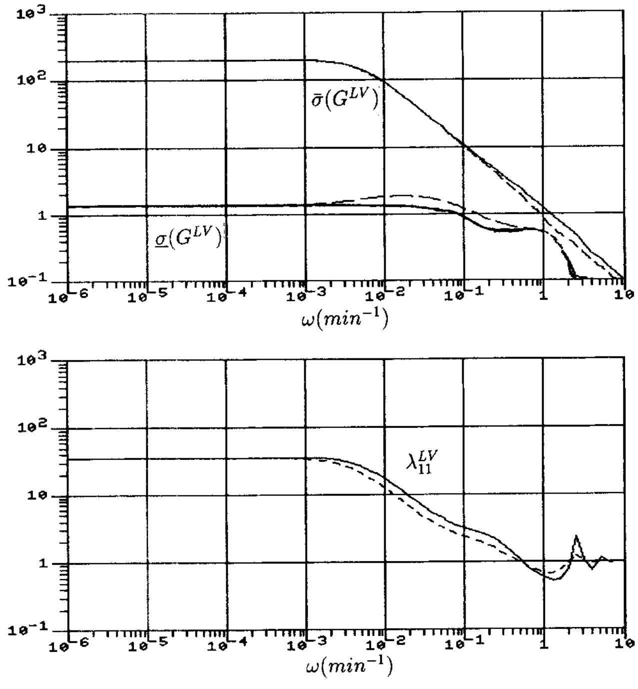

Figure 3. Column A: Singular values and $|\lambda_{11}|$ as a function of frequency.
— full-order model; ---- simplified model (8)

constant, will immediately yield a corresponding increase in L. This increase will yield a corresponding increase in V, which subsequently will increase L even more, and so on. Consequently, the effect is that the internal flows eventually will approach infinity.

Example columns

Accuracy of Simplified Model for Column A. To illustrate the accuracy of the simplified model (8), in Figure 3 we compare the singular values and RGA values as a function of frequency with those for the linearized full-order model with 82 states. From Figure 3 we conclude that the simplified model seems to capture the true behavior quite well. The other columns yield similar results, except for columns E and F where there are some differences at high frequency.

Different Configurations for Column A. Figure 4 shows the gain elements g_{ij} as a function of frequency for four configurations.

RGA Values for Columns A–G. Figure 5 shows the 1,1 element of the RGA as a function of frequency for five different configurations. We shall discuss these values in more detail later. Figure 5 should be compared with similar figures given in Skogestad and Morari (1988a) for the case with no flow dynamics, that is, $dL_B = dL$. The introduction of flow dynamics is seen to change the results dramatically. First, the response becomes decoupled at high frequency, which makes the RGA

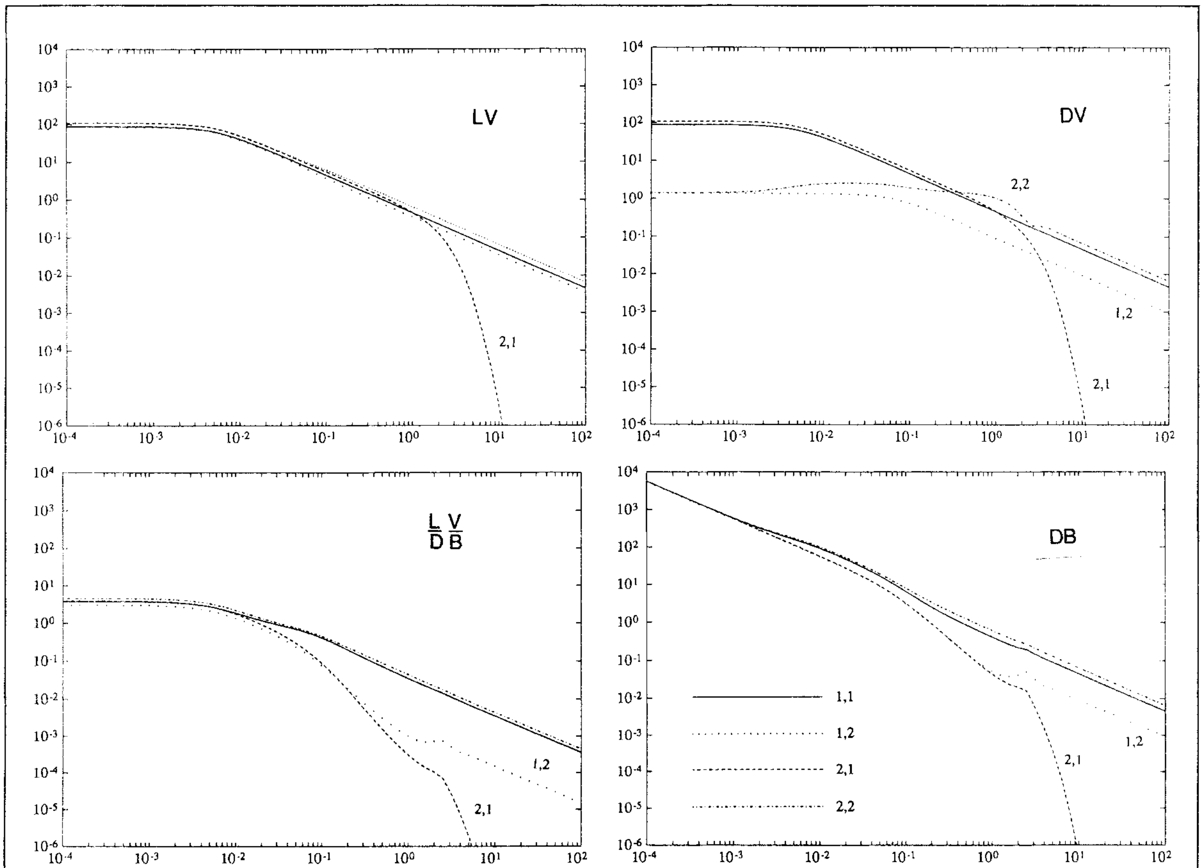

Figure 4. Column A: Gain elements $|g_{ij}|$ as a function of frequency, using four different configurations.

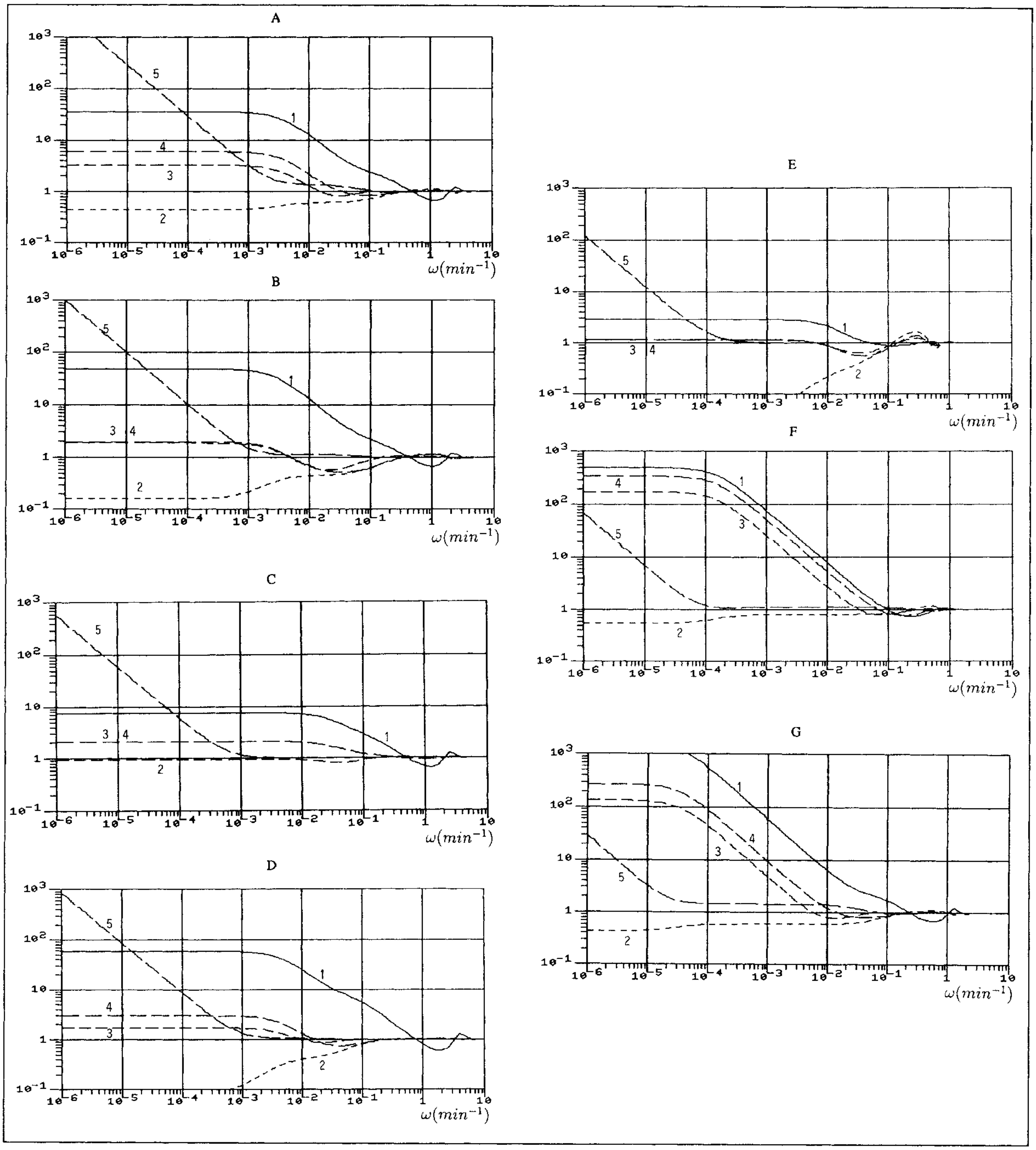

Figure 5. Columns A–G: $|\lambda_{11}|$ as a function of frequency, using five different configurations.
1. LV; 2. DV; 3. (L/D)(V/B); 4. (L/D)V; 5. DB

matrix equal to identity. Second, the importance of the time constant τ_2 associated with internal flows is seen to be much less.

Level and Pressure Control

This paper does not treat this important problem in any detail. To get a fast initial response one will usually prefer to use V or B, or a combination of these, to control reboiler level, and to use L, D, or V_T (or a combination) to control condenser level; similar arguments also apply to single-loop composition control. Level control is for obvious reasons simplest if one uses a large flow. For example, when the reflux is large it is often difficult to use D for level control.

Open-loop (Manual) Operation

The term "open loop" should here be put in quotes because we are not talking about an uncontrolled column, but assume that levels and pressure are perfectly controlled and consider the effect of the remaining independent variables on the compositions. For example, open-loop operation of the LV configuration assumes levels and pressure to be perfectly controlled with D, B, and V_T, and the remaining manipulated variables, L and V, to be constant (in manual). (The development of a true open-loop 5 × 5 model where all five flows, L, V, D, B, and V_T, are independent variables is considered by Skogestad, 1989.)

Good open-loop performance is achieved if the effect of disturbances on compositions is small, that is, if the built-in disturbance rejection or self-regulation is good. Disturbances include changes in feed conditions (F, z_F, q_F) and disturbances on all flows, L, V, D, B, and V_T.

The steady state effect of disturbances on compositions may be evaluated quite accurately by considering the steady state effect of the disturbances on D/B or equivalently D/F. This follows since the major effect on product compositions of any change in the column may be obtained by assuming the separation factor S to be constant (Shinskey, 1984). Differentiating the component material balance with S constant yields

$$\frac{dy_D}{(1-y_D)y_D} = \frac{dx_B}{(1-x_B)x_B} = \frac{F}{I_S}dz_F - \frac{y_D - x_B}{I_S}Fd\left(\frac{D}{F}\right) \quad (16)$$

where the impurity sum is defined as $I_S = D(1-y_D)\,y_D + B\,(1-x_B)x_B$. That is, compositions y_D and x_B are mainly affected by changes in z_F and D/F. The effect of various flow disturbances on $Fd(D/F)$ is given in Table 3. Configurations with small entries in Table 3 are insensitive to disturbances at steady state and should be preferred. We see that the (L/D)(V/B) configuration is a good choice when the reflex is large. Note that the DB configuration is not included in Table 3 because D and B may not be specified independently at steady state. In general, the values obtained with the above approximation are quite accurate, in particular, for the gain of the least pure product. For example, the effect of a change in feed rate on y_D with constant L and V from Eq. 16 is

$$\left(\frac{\partial y_D}{\partial F}\right)_{L,V} \approx -\frac{(1-y_D)y_D(y_D-x_B)}{D(1-y_D)y_D + B(1-x_B)x_B}F\left(\frac{\partial D/F}{\partial F}\right)_{L,V} \quad (17)$$

where from Table 3,

$$F\left(\frac{\partial D/F}{\partial F}\right)_{L,V} = 1 - q_F - D/F.$$

For Column A the value of $\partial y_D/\partial F_{L,V}$ obtained by this approximation is 0.49, which compares quite well with the exact value of 0.394. For column C, which has an unpure top product, the values are 0.882 and 0.883.

Another interesting point is that the values in Table 3 for disturbance rejection happen to correlate closely with the RGA for most configurations (Skogestad, 1988). This is one of the fortunate circumstances that make the RGA useful for distillation configuration selection.

The differences between the configurations with respect to disturbance sensitivity are smaller at high frequency. Consider Figure 6, which shows the effect of a disturbance in F on the compositions for column D for three different configurations. Although the gain is entirely different at steady state—infinity for the DB configuration, zero for the (L/D)(V/B) configuration—the high-frequency gain (initial effect on compositions) is quite similar.

Table 3. Open-Loop Sensitivity to Flow Disturbances, $F(\partial(D/F)\partial d)_{u_1,u_2}$

Configuration u_1u_2	Disturbance d: dF	dv_d	dD_d	dB_d
$LV, \frac{L}{V}V, \frac{L}{V}L$	$1 - q_F + \epsilon_F - D/F$	1	0	0
$DX, \frac{D}{X}X$	$-D/F$	0	1	0
$BX, \frac{B}{X}X$	B/F	0	0	-1
$\frac{D}{B}X$	0	0	B/F	$-D/F$
$\frac{L}{D}V, \frac{D}{L+D}V$	$-\frac{V'/F}{1+L'/D}$	$\frac{1}{1+L'/D}$	$\frac{L'/D}{1+L'/D}$	0
$L\frac{V}{B}, L\frac{B}{V+B}$	$\frac{L'/F}{1+V'/B}$	$\frac{1}{1+V'/B}$	0	$-\frac{V'/B}{1+V'/B}$
$\frac{L}{D}\frac{V}{B}, \frac{D}{L+D}\frac{V}{B}$	0	$\frac{1}{r'}$	$\frac{L'/D}{r'}$	$\frac{-V'/B}{r'}$

Applies to steady state
$dv_d = (1-\epsilon_V)dV_d - (1-\epsilon_L)dL_d$

ϵ_V, ϵ_L, and ϵ_F represent deviations from constant molar flows: $dD = -(1-\epsilon_L)dL + (1-\epsilon_V)dV + (1-q_F+\epsilon_F)dF$
X = any other manipulated input u except D, B and D/B
Subscript d denotes an additive disturbance on this flow
$V' = (1-\epsilon_V)V$; $L' = (1-\epsilon_L)L$; $r' = 1 + L'/D + V'/B$

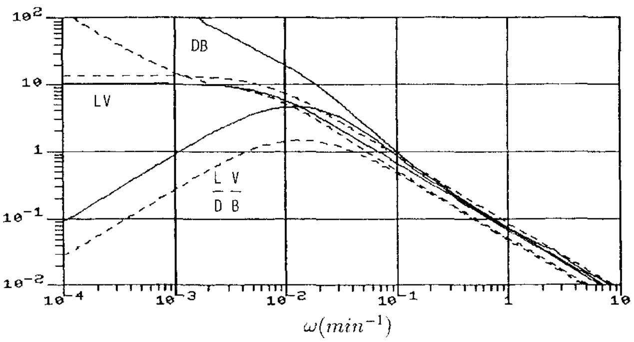

Figure 6. Column D: Open-loop effect of disturbance in F on compositions, using three different configurations.

— $\Delta y_D^S/\Delta F$; ---- $\Delta x_B^S/\Delta F$

One-Point Composition Control

For one-point control we shall introduce the following convention: The u_1u_2 configuration means that u_1 is used for composition control and u_2 is constant (in manual).

Most industrial columns use one-point control, usually of top composition; that is, y_D is controlled by u_1. The composition in the other end of the column is then left uncontrolled or is possibly slowly adjusted by changing u_2 manually. The automatic control of the one composition is quite simple, and the choice of u_1 is usually not critical, although reflux L or boilup V is usually used because they directly affect compositions. The more important choice is which manipulated variable u_2 is left in manual, because this affects the behavior of the uncontrolled composition.

One should never choose u_2 to be D or B because this locks the material balance and makes it impossible to adjust D/F as desired in the case of disturbances in feed rate and feed composition, Eq. 16. Acceptable choices for u_2 are, for example, $L, V, L/D$, and V/B. Because of the strong coupling between the top and the bottom of the column, one generally finds that controlling the one composition also gives reasonably good control of the uncontrolled composition. Unfortunately, there is no simple method (like Table 3 for open loop) to find the best choice for u_2 However, based on a large number of numerical evaluations of $(\partial_{yD}/\partial d)_{B,u_2}$ and $(\partial \chi_B/\partial d)_{yD,u_2}$ for various disturbances d we found the best choice for u_2 to be L or V if disturbances in z_F are considered. Data for a typical example are shown in Table 4. Using a ratio, for example, $u_2 = L/D$ or V/B, is of course best for disturbances in F. However, if the feed rate is measured then using feedforward action by choosing u_2 as L/F (possibly with a dynamic lag in the feedforward loop) or V/F may be almost as good.

In many cases the reason for using one-point control is that the column is operating at maximum capacity. In this case u_2 is not free to choose and will be equal to the flow that is limited (for example, V if boilup or vapor flows are limiting, V_T if cooling is limiting). Again, one should not operate the column such that D or B are at their maximum values.

The conclusion is that the LV or VL configuration (possibly with a feedforward action from F to u_2) is usually the best choice for one-point composition control. This is probably why this configuration is preferred in industry (however, as we shall see below the LV configuration is not the best for two-point control). If reflux or boilup is large, then *level control* with D or B may be

Table 4. Sensitivity to Disturbances with One-Point Control of y_D for Column A, $(\partial x_B/\partial d)_{y_D,u_2}$

Input	Disturbance d		
u_2*	F	z_F	V_d
L	0.086	0.001	0
V	0.100	0.033	−0.031
D	0.980	2.000	0
B	−0.980	2.000	0
L/D	0	−0.191	0
V/B	0	0.216	−0.028
D/B	0	2.000	0

*u_2 in manual

difficult. In this case the DV or BL configurations or a ratio configuration may be preferred for one-point control.

Two-Point Composition Control

For two-point control we use the following convention: Single-loop control with the u_1u_2 configuration means that u_1 is used to control y_D and u_2 to contrtol x_B.

Frequency-dependent RGA

The magnitude of the 1,1 element of the RGA, $|\lambda_{11}|$, is shown as a function of frequency in Figure 5. Usually, only the steady state value of the RGA is considered (Shinskey, 1984). However, it has been suggested that the frequency-dependent RGA may yield additional information (McAvoy, 1983) and newer results support this. Skogestad and Morari (1987c) show that large RGA values indicate a plant that is very sensitive to element-by-element uncertainty, and even more importantly to input uncertainty, and therefore single-loop controllers should be used. Nett (1987) has demonstrated that RGA values close to 1 at frequencies corresponding to the closed-loop bandwidth means that single-loop controllers may be designed independently. Therefore, a column will be easy to control with single loops if the RGA is close to 1 in the frequency range of about 0.01 to 1 min^{-1}. This statement will be the basis for the discussion that follows. Because of the liquid flow dynamics, the plant will be triangular at high frequency and the RGA will approach 1 for all configurations. As one measure of how easy a configuration is to control we shall consider the frequency, ω_1, where the RGA approaches 1. Small values of ω_1 are preferred. We shall see that for a given configuration there is no direct relationship between this value and the steady state RGA. However for *comparing* configurations there is a relationship in some cases, Eqs. 19 and 20, below. This provides some justification for using the steady state RGA. The results in Figure 5 are next discussed for each configuration.

LV Configuration. λ_{11} stays at its steady state value until it reaches the first corner frequency, where it starts falling off with a slope of −1 on the logarithmic plot. Taking a close look, for columns A,B,D, and G one can observe a small intermediate region where the slope is somewhat less than −1. This is the effect of the time constant τ_2. λ_{11} then continues falling off with a −1 slope down to unity. For all the columns considered the frequency at which it reaches unity is

$$\omega_1^{LV} \approx 1/\theta_L \tag{18}$$

which is the frequency at which the response becomes decoupled, Eq. 8. Note that increasing θ_L makes the decoupling take place at a lower frequency and is thereby beneficial for control purposes. Both for tray and packed columns the largest value obtainable for θ_L is about $0.6M_I/L$ where M_I is the total holdup inside the column (Skogestad and Morari, 1988a). That is, θ_L is increased by increasing the liquid holdup inside the column. Note that there is no relationship between θ_L(or ω_1) and the steady state RGA. For example, columns A and G have $\lambda_{11}^{LV}(0)$ values of 35 and 1,673, respectively, but θ_L only differs by a factor 2, equal to their difference in liquid holdup. [On the other hand, it may be shown that for most well-designed columns we have $\tau_1 \approx \lambda_{11}^{LV}(0)\theta_L$ (combine Eqs. 39, 47, 48, and 66 in Skogestad and Morari, 1988a); that is, there is a direct relationship between $\lambda_{11}^{LV}(0)$ and the dominant time constant τ_1.]

(L/D)V Configuration. From Figure 5 we observe that the shape of the RGA curve for this configuration follows that of the LV configuration and that the frequency for crossing with 1 is reduced by a factor corresponding to the ratio between their steady state RGA values, that is

$$\omega_1^{(L/D)V} \approx \omega_1^{LV} \frac{\lambda_{11}^{(L/D)V}(0)}{\lambda_{11}^{LV}(0)} \tag{19}$$

where $\lambda_{11}(0)$ is the steady state RGA-value.

(L/D)(V/B) Configuration. The same arguments as above apply to this configuration and we have

$$\omega_1^{(L/D)(V/B)} \approx \omega_1^{LV} \frac{\lambda_{11}^{(L/D)(V/B)}(0)}{\lambda_{11}^{LV}(0)} \tag{20}$$

The fact that the shape of the λ_{11} curve is similar for the LV, *(L/D) V*, and *(L/B) (V/B)* configurations, assures one that selecting the one with lowest λ_{11} value at steady state guarantees the same to hold under dynamic conditions. Simple methods for estimating $\lambda_{11}(0)$ for various configurations are presented by Shinskey (1984) and Skogestad and Morari (1987b).

DV Configuration. $|\lambda_{11}|$ for the DV configuration shows a different behavior than for the three discussed above. The main difference is of course that this configuration always has $\lambda_{11}(0)$ less than 1. We have

$$\lambda_{11}^{DV}(0) \approx \left[1 + \frac{Bx_B}{D(1 - y_D)}\right]^{-1}$$

which is close to one for columns with a pure bottom product and close to zero when the top is pure. The RGA value becomes 1 at the same frequency as for the LV configuration, that is,

$$\omega_1^{DV} \approx \omega_1^{LV} \tag{21}$$

(for columns with a pure top product the RGA is close to 1 at all frequencies and ω_1^{DV} is not too meaningful). One disadvantage with the DV configuration for two-point control is that the RGA values, and therefore control behavior, may depend strongly on the operating conditions.

DB Configuration. The RGA for the DB configuration is infinite at steady state. It falls off with a -1 slope and the low-frequency asymptote crosses 1 at (Skogestad et al., 1990)

$$\omega_1^{DB} \approx \omega_1^{LV} \frac{1}{1/(Bx_B) + 1/[D(1 - y_D)]} \frac{\ln S}{L_B}. \tag{22}$$

The term multiplying ω_1^{LV} is typically of the order 0.01. It is much less than 1 for columns with a high-purity product and for columns with large reflux. Thus, although the RGA value for the DB configuration is much worse (higher) than for the LV configuration at low frequency, it is significantly better (closer to 1) in the frequency range important for feedback control, that is, from about 0.01 to 1 min^{-1}. The observation of Finco et al. (1989) that the DB configuration gives better control performance than the LV configuration for a propane-propylene column (similar to column D) is therefore not surprising from the RGA values. In fact, from the RGA values all the example columns seem to be quite easy to control using the DB configuration.

Comparison of control performance

In this section we compare different configurations based on their achievable control performance using single-loop PI controllers for composition control. Single-loop controllers are chosen because this is the preferred controller structure in industry.

The controllers were tuned to optimize robust performance using the same weights for performance and uncertainty as used by Skogestad and Morari (1988b). The uncertainty weight allows 20% error on each manipulated input and a variable time delay of up to 1 min. The performance objective is that the magnitude of the worst-case sensitivity function, $(I + GC)^{-1}$, should be smaller than a given upper bound. This upper bound prescribes a closed-loop time constant τ_B better than 20 min, and an amplification of high-frequency disturbances of less than 2. Mathematically, the optimal PI tunings were obtained by minimizing the structured singular value, μ, as discussed in more detail by Skogestad and Morari (1988b). A μ value less than 1 means that the performance bound is satisfied for the worst case, and smaller values imply a larger margin to the bound. Note that the μ values obtained here are indicative of the feedback properties only (set-point changes), and the results do not take into account that some configurations are less sensitive to disturbances than others; (see Figure 6).

The minimized μ values for single-loop PI control of the seven columns using the LV, DV, (L/D)(V/B), and DB configurations are summarized in Table 5. The results show that the (L/D)(V/B) configuration is the best, with a μ value less than 1 for all columns. Note that the difference in control performance is generally larger than one might expect from comparing the μ values in Table 5. For example, for column A the value of μ for the four configurations is 0.94, 0.84, 0.67, and 0.77, respectively. However, if we compute the achievable closed-loop time constant, τ_B, by adjusting the performance weight such that the optimal μ value is 1, we find that τ_B is 14.4, 8.2, 3.2, and 4.4 min, respectively. Consequently, the achievable response time for the LV configuration is almost five times larger (poorer) than for the (L/D)(V/B) configuration.

The μ values in Table 5 correlate very well with the RGA values at high frequency (that is, $0.01 < \omega < 1$ min^{-1}). For example, the LV configuration has the worst performance (high μ value) for almost all columns, and this is confirmed by the large RGA values. The only exception is column E, where the LV configuration has the lowest μ value, but its RGA is also very low for this column, less than 2.8 at all frequencies. Note that this column is usually considered easy to control because of low reflux and because one of the products is relatively unpure. From

Table 5. Optimal μ Values with PI Control

Column	Configuration			
	LV	*DV*	*(L/D)(V/B)*	*DB*
A	0.94	0.84	0.67	0.77
B	0.94	0.81	0.75	0.80
C	1.03	0.71	0.69	0.73
D	1.07	0.87	0.66	0.70
E	0.80	1.07	0.97	1.12
F	0.86	0.80	0.73	0.71
G	0.92	0.85	0.66	0.75

the μ values the DV configuration is poor for column E and good for column C; also, this agrees with the RGA.

Of course, there is not an exact match between μ and the RGA values. The μ values indicate that the DB configuration is generally somewhat poorer than the (L/D)(V/B) configuration. This is not possible to infer from the RGA values at high frequencies, which are close to 1 for both these configurations. The only exception is column F, where the RGA at intermediate frequencies ($\omega < 0.01$ min^{-1}) is clearly better for the DB configuration, and indeed for this column the μ value proves to be lower for the DB than for the (L/D)(V/B) configuration.

The effect on μ of adding a measurement delay of 5 min for columns A and D is shown in Table 6. We note that the LV configuration shows the largest increase in μ values. Again, this may be expected from the RGA plots: The additional delay forces the bandwidth to be smaller. This means that the interactions (RGA values) at lower frequencies become more important. This is detrimental for the LV configuration, where the RGA values increase sharply at lower frequencies.

Simulations

The μ values in Table 5 form the basis for comparison, and the simulations are included mainly to visualize the differences between the configurations. Nonlinear simulations for column A and D, a C3 splitter, are shown in Figures 7 and 8. Responses are shown for a 30% increase in feed rate F at $t = 0$ and a 20% increase in feed composition z_F at $t = 50$ min using the PI controllers given in Table 7. A 1 min time delay for each input is included. The simulations support the conclusions made above with respect to differences between the configurations.

The PI tunings used in these simulations are not the same as those that formed the basis for the μ values in Table 5. The reason is that the μ values were obtained by considering set-point changes, and the tunings are somewhat sluggish for disturbance rejection. The values of k_y, k_x, τ_{Iy}, and τ_{Ix} in Table 7 were obtained as follows: Maximize the ratio k/τ_I such that the worst-case (with respect to uncertainty) peak of $\bar{\sigma}\,[(I + GC)^{-1}(j\omega)]$ is less than 2. This maximizes the controller gain at low and intermediate frequencies, where large gains are desired to reject the disturbances, but guarantees a reasonable robustness margin. For simplicity, k/τ_I is assumed equal in the two loops. These responses may not be as fast in practice. If holdups and time delay were increased by a factor of 5 (M_i/F = 2.5 min, θ = 5 min), times in Table 7 and Figures 7 and 8 would increase by a factor of 5.

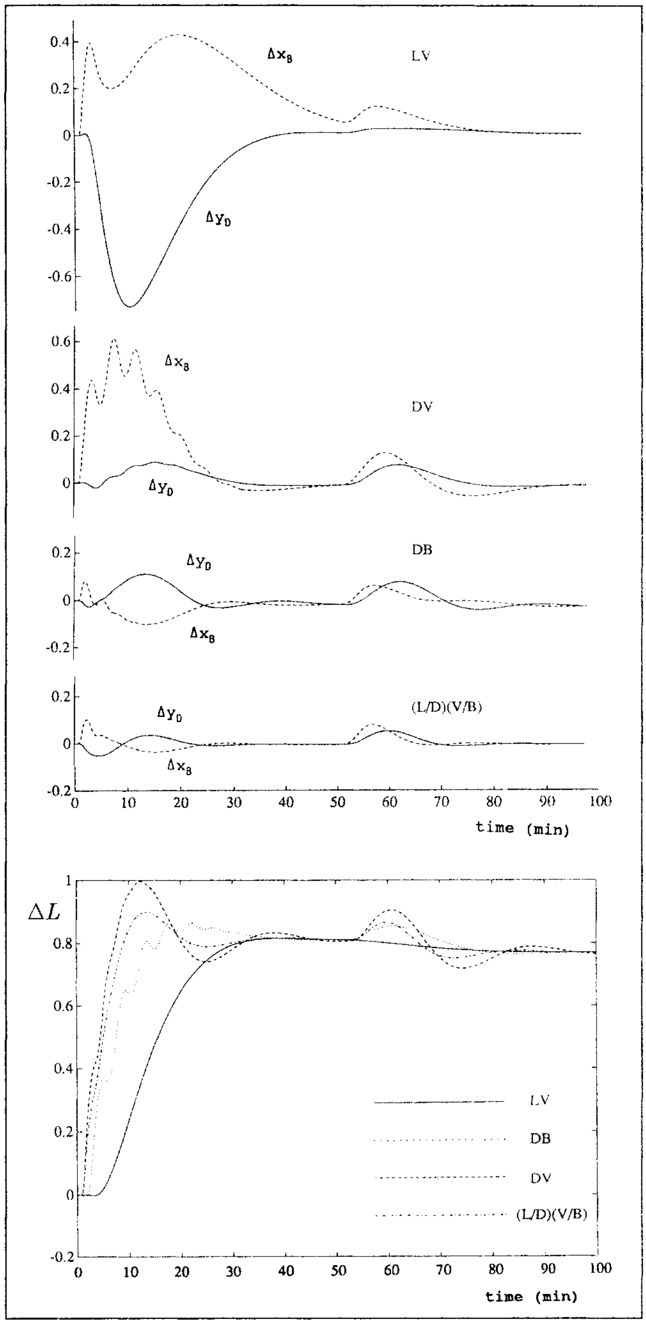

Figure 7. Column A responses to a 30% increase in *F* at $t = 0$ and a 20% increase in z_F at $t = 50$ min.

Top: Compositions — $\Delta y_D^S(t)$; ---- $\Delta x_B^S(t)$
Bottom: Reflux rate L.

Table 6. Optimal μ Values with PI Control and 5 min Analyzer Delay

Column	Configuration			
	LV	*DV*	*(L/D)(V/B)*	*DB*
A	1.65	1.24	1.02	1.24
D	1.82	1.17	1.01	1.15

Discussion

The most important industrially used configuration not studied in this paper is probably the LB configuration. It is in many

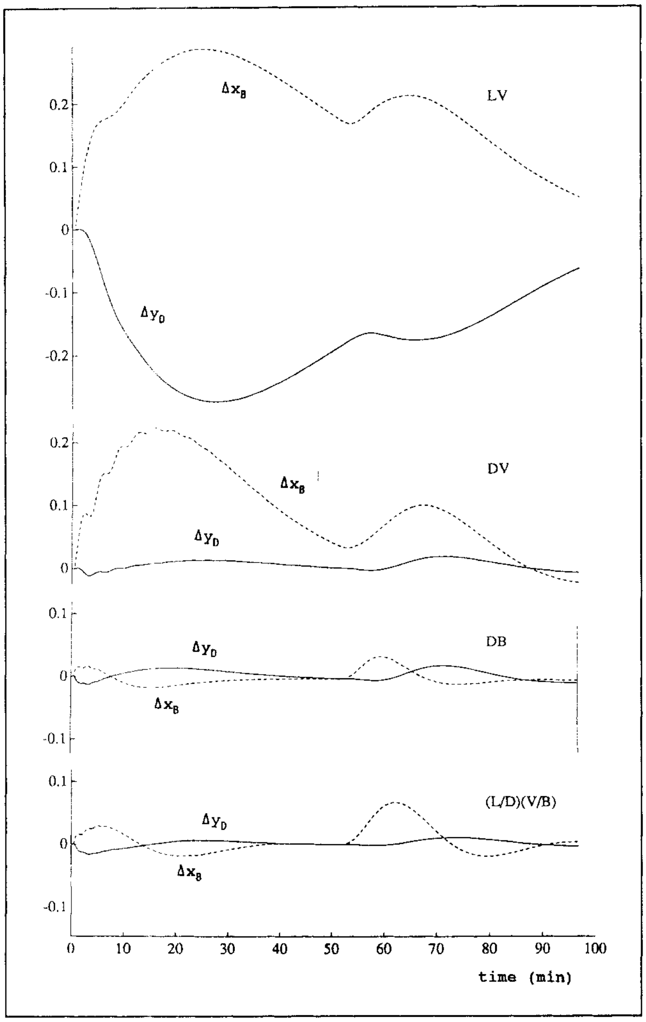

Figure 8. Column D: Responses to a 30% increase in F at $t = 0$ and a 20% increase in z_F at $t = 50$ min.
— $\Delta y_D^S(t)$; ---- $\Delta x_B^S(t)$

respects similar to the DV configuration, but while the DV configuration is best for two-point control when the bottom product is pure, the LB scheme is preferred when the top product is pure. With perfect level control configurations involving, for example, $L/(L + D)$ or $D/(L + D)$ are equivalent to using L/D (except for a gain change in the controller).

Disturbances

The RGA and μ values used here do not consider disturbances and are indicative of the "pure" feedback properties only. Disturbances are usually the main reason for using feedback for distillation columns, and as we have shown their effect depends on the choice of configuration. To eealuate their effect one might minimize μ with disturbances included in the performance objective, use other simpler tools such as the open-loop frequency response of the disturbances, Figure 6, or the relative disturbance gain (RDG, Stanley et al., 1985). The frequency-dependent RDG gives the effect on the outputs of disturbances under feedback relative to the effect under open loop (Skogestad and Morari, 1987d). Nevertheless, since the disturbance rejection is based on feedback, we do not expect drastic changes in the conclusions with respect to which configuration to prefer if disturbances are included in the analysis, as confirmed by the simulations in Figures 7 and 8.

Holdups, measurements, and dead time

We assumed that product compositions y_D and x_B are measured with a delay of up to 1 min. This may seem somewhat unrealistic. However, in a practical situation one may use temperatures to estimate compositions, and update the estimates using a cascade from the analyzers. As to the question of using very low holdups in the condenser and reboiler it is usually best to neglect or use small values for these holdups for the two reasons:

1. If temperatures are used to estimate compositions, then the dynamic response inside the column matters, and this response is usually weakly affected by the reboiler and condenser holdups (Skogestad and Morari, 1988a).
2. The holdups (levels) in the condenser and reboiler may vary with time and it is then safest (from a robustness viewpoint) to use the smallest holdup when designing the controllers (Skogestad and Morari, 1988b).

Perfect level control assumed makes almost no difference for the LV configuration, but for others the composition performance may deteriorate if the level loops are poorly tuned. The key issue is not that the level is tightly controlled, but responses for the internal flows, L_T and V_B, are similar to those with perfect level control. Implementing the $(L/D)(V/B)$ configuration in Figure 2 may not be used in practice. The level controller should set $L + D$ rather than D. A simple analysis shows that this makes the response in L (and y_D) to a change in L/D much less sensitive to the level tuning.

Table 7. PI Settings for Columns A and D Used in Simulations; $C(s) = k^{(1+\tau_I s)/(\tau_I s)}$

Column	Config.	k_y	k_x	τ_{Iy} min	τ_{Ix} min
A	*LV*	0.0823	0.817	1.18	11.7
	DV	1.06	0.610	8.83	5.08
	$(L/D)(V/B)$	12.4	12.2	4.13	4.07
	DB	0.938	1.04	7.22	8.04
D	*LV*	1.76	0.961	7.38	4.00
	DV	2.88	3.58	5.77	7.17
	$(L/D)(V/B)$	113	113	4.19	4.19
	DB	3.53	2.61	5.89	4.35

Gains k_y and k_x are for scaled compositions, Eq. 1.

Conclusions

The results in this paper are summarized below. The arguments mainly refer to composition control, although comments on level control are included. The arguments regarding two-point control refer to columns that are difficult to control with the conventional LV scheme, that is, $\lambda_{11}^{LV}(0) > 5$.

LV Configuration. A good choice for one-point control, but not recommended for two-point control because of sensitivity to disturbances, Table 3, and poor control performance due to interactions between control loops. In particular, the LV configuration performs poorly with large dead times.

DV Configuration. One-point control: D must *always* be used for automatic control (never in manual). May be better than LV for columns with large reflux because top level control is simpler. Two-point control: Works relatively poor when bottom product is not purer than top, but is better when bottom product is pure (e.g., column C). Disadvantages are: 1. performance may change depending on operating conditions; 2. very poor performance if failure leads to D constant (for example, measurement in top fails).

DB Configuration. Unacceptable performance if used for one-point control. Two-point control: Good control quality, in particular for columns with high purity and/or large reflux. Simple to implement. Level control also favors this for columns with large reflux. The main disadvantage is that it lacks integrity; performance is very poor if failure gives D or B constant. In particular, one cannot put one of the loops in manual.

(L/D)(V/B) Configuration. Overall this is the best choice for all modes of operation. The main disadvantage is the need for measurements of all flows, L, D, B, and V, which makes it more failure sensitive and more difficult to implement.

(L/D)V Configuration. Behaves somewhere between LV and (L/D)(V/B) configurations. For two-point control a plot of the frequency-dependent RGA is very useful to evaluate expected control performance. A good approximation of the important high-frequency behavior is obtained using Eqs. 18–22.

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Notation

B = bottom product, kmol/min
$C(s)$ = transfer function for controller
D = distillate (top product), kmol/min
F = feed rate, kmol/min
$G(s)$ = transfer matrix for column
$L \equiv L_T$ = reflux flow rate, kmol/min
L_B = liquid flow rate into reboiler, kmol/min
M_i = liquid holdup on theoretical tray i, kmol
$M_I = (N-1)M_i$ = total holdup inside column, kmol
N = number of theoretical stages in column
q_F = fraction liquid in feed
RGA = relative gain array matrix, elements are λ_{ij}
$S = y_D(1-x_B)/(1-y_D)x_B$ = separation factor
$V \equiv V_B$ = boilup from reboiler, determined by heating, kmol/min
V_T = vapor flow rate above top tray, determined by cooling, kmol/min
x_B = mole fraction of light component in bottom product
y_D = mole fraction of light component in distillate (top product)
z_F = mole fraction of light component in feed

Greek letters

$\alpha = y_i(1-x_i)/[x_i(1-y_i)]$ = relative volatility
$\lambda_{11}(s) = (1 - g_{12}(s)g_{21}(s)/g_{11}(s)g_{22}(s))^{-1}$ 1,1 element in RGA
μ = peak value of structured singular value for robost performance
ω = frequency, min^{-1}
$\bar{\sigma}(A), \underline{\sigma}(A)$ = maximum and minimum singular values of matrix A
τ_1 = dominant time constant for external flows, min
τ_2 = time constant for internal flows, min
τ_I = integral time for PI controler, min
$\tau_L = (\partial M_i/\partial L)_V$ = hydraulic time constant, min
$\theta_L = (N-1)\tau_L$ = overall lag for liquid response, min

Superscript

S = scaled composition, Eq.1

Literature Cited

Birky, G., T. McAvoy, and B. Tyreus, "DICODE—An Expert System for Designing Distillation Column Controls," *Proc. Am. Control Conf. (ACC)*, Pittsburgh, 95 (1989).

Finco, M. V., W. L. Luyben, and R. E. Polleck, "Control of Distillation Columns with Low Relative Volatility," *Ind. Eng. Chem. Res.*, **28**, 76 (1989).

Häggblom, K. E., and K. V. Waller, "Transformations and Consistency Relations of Distillation Control Structures," *AIChE J.*, **34**, 10, 1634 (1988).

Luyben, W. L., "Introduction and Overview of Distillation Column Control" (discussion), *Industrial Process Control*, AIChE, New York (1979).

——— "The Impact of Process Diversity on Distillation Column Control," *Prepr. IFAC Symp. DYCORD+'89*, Maastricht, NL (1989).

McAvoy, T. J., *Interaction Analysis*, Instrument Soc. of America, Triangle Park, NC (1983).

McCune, L. C., and P. W. Gallier, "Digital Simulation: A Tool for the Analysis and Design of Distillation Controls," *ISA Trans.*, **12**(3), 193 (1973).

Nett, C. N., Paper presented at Am. Control Conf., Minneapolis (June, 1987).

Shinskey, F. G., *Process Control Systems*, McGraw-Hill, New York (1967).

———*Distillation Control*, 2d ed., McGraw-Hill, New York (1984).

Skogestad, S., 1988, "Disturbance Rejection in Distillation Columns," *Proc. CHEMDATA 88*, 19th *Eur. Symp.* Computer Applications in Chem. Industry, Göteborg, Sweden, 365 (1988).

———"Modelling and Control of Distillation Columns as a 5 × 5 system," 20th Eur. Symp. Computer Applications in Chem. Industry (Cachi 89), Erlangen, BRD, *Dechema Monographs*, **116**, 403 (1989).

Skogestad, S., and M. Morari, "Control Configuration Selection for Distillation Columns," *AIChE J.* **33**,(10), 1620 (1987a).

———, "A Systematic Approach to Distillation Column Control," Symp. Distillation 87, Brighton, *Ind. Chem. Eng. Symp. Ser.*, **104**, A71 (1987b).

———, "Implication of Large RGA Elements on Control Performance," *Ind. Eng. Chem. Res.*, **26**,(11), 2121 (1987c).

———, "Effect of Disturbance Directions on Closed-loop Performance," *Ind. Eng. Chem. Res.*, **26** (10), 2029 (1987d).

———, "Understanding the Dynamic Behavior of Distillation Columns," *Ind. Eng. Chem. Res.*, **27**(10), 1848 (1988a).

———, "LV Control of a High-purity Distillation Column," *Chem. Eng. Sci.*, **43**,(1), 33 (1988b).

Skogestad, S., E. W. Jacobsen, and M. Morari, "DB Control of Distillation Columns," *Ind. Eng. Chem. Res.*, (1990).

Stanley, G., M. Marino-Galarraga, and T. J. McAvoy, "Shortcut Operability Analysis. 1: The Relative Disturbance Gain," *Ind. Eng. Chem. Process Des. Dev.*, **24**(4), 1181 (1985).

Takamatsu, T., I. Hashimoto, and Y. Hashimoto, "Selection of Manipulated Variables to Minimize Interaction in Multivariate Control of a Distillation Column," *Int. Chem. Eng.*, **27**(4), 669 (1987).

Waller, K., "Distillation Control System Structures," *Prepr. IFAC Symposium DYCORD 86*, Bournemouth, UK (1986).

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